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ON THRESHOLD GRAPHS AND REALIZATIONS OF GRAPHICAL PARTITIONS

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A triple of vertices (x, v, y) in a graph $G = (V, E)$ such that $xv \in E$ and $vy \notin E$ is called lifting if $\deg(x) \le \deg(y)$ and lowering if $\deg(x) \ge 2 + \deg(y)$. A lowering rotation of an edge in a graph G corresponding to a lowering triple (x, v, y) is a transformation of this graph that replaces the edge xv by the edge vy. We prove that G is a threshold graph if and only if it has no lifting triples of vertices. This result has three corollaries:

1. The graphical partition corresponding to G is a maximal graphical partition if and only if G is a threshold graph.

2. An arbitrary partition λ is a maximal graphical partition if and only if the head of λ is equal to its tail.

3. Each realization of an arbitrary graphical partition μ can be obtained by a finite sequence of lowering rotations of edges from a threshold realization of an appropriate maximal graphical partition λ such that $\lambda \ge \mu$.

Keywords: graph, threshold graph, lattice, integer partition, graphical partition, Ferrers diagram.

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