

MSC: 45B05, 45Q05

DOI: 10.21538/0134-4889-2017-23-2-210-219

**ON THE APPROXIMATE SOLUTION OF AN INVERSE BOUNDARY VALUE
PROBLEM BY THE METHOD OF FINITE-DIMENSIONAL APPROXIMATION
OF THE REGULARIZED SOLUTION**

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We solve the inverse boundary value problem for the heat equation. The problem is reduced to an integral equation of the first kind, which in turn is reduced to a finite-dimensional equation by means of discretization in two variables. The latter equation is solved by means of A. N. Tikhonov's regularization method with the regularization parameter chosen according to the residual principle with discretization error taken into account. It is shown that the problem does not satisfy V. K. Ivanov's condition, which would allow to employ the modulus of continuity of the inverse operator. That is why, to estimate the error of the approximate solution, we propose a numerical approach using the discretization of the problem. The obtained estimate is compared with the classical estimate in terms of the modulus of continuity. The approach proposed in this paper makes it possible to considerably extend the class of problems to which it is applicable.

Keywords: ill-posed problem, integral equation, estimation of error, regularizing algorithm, finite-dimensional approximation.

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doi: 10.1016/0041-5553(69)90005-6.

The paper was received by the Editorial Office on December 10, 2015.

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Cite this article as:

A. I. Sidikova, On the approximate solution of an inverse boundary value problem by the method of finite-dimensional approximation of the regularized solution, *Trudy Inst. Mat. Mekh. UrO RAN*, 2017, vol. 23, no. 2, pp. 210–219.