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ON THE ASYMPTOTICS OF A SOLUTION TO A SECOND-ORDER ELLIPTIC
EQUATION WITH A SMALL PARAMETER
AND A PIECEWISE SMOOTH BOUNDARY FUNCTION

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We study the asymptotic behavior of a solution to the first boundary value problem for a second-order elliptic equation in the case when the small parameter is a factor at only one of the highest derivatives and the limit equation is an ordinary differential equation. We consider the case when the boundary function is piecewise smooth. Moreover, the point where the smoothness is violated is a point of jump discontinuity and coincides with the point where a characteristic of the limit equation touches the boundary from inside. Although the limit equation has the same order as the original equation, the problem under consideration is bisingular. We investigate the asymptotic behavior of the solution to this problem using the method of matched asymptotic expansions.

Keywords: singular problems, boundary value problems for partial differential equations, asymptotic expansions, matching method.

REFERENCES

1. Vishik M.I., Lyusternik L.A. Regular degeneration and boundary layer for linear differential equations with small parameter. *Uspechi Mat. Nauk*, 1957, vol. 12, no. 5(77), pp. 3–122 (in Russian).
2. Trenogin V.A. The development and applications of the asymptotic method of Lyusternik and Vishik. *Russian Math. Surveys*, 1970, vol. 25, no. 4, pp. 119–156. doi: 10.1070/RM1970v025n04ABEH001262.
3. Nayfeh Ali H. *Perturbation methods*. New York, John Wiley and Sons, 1973, 425 p. ISBN: 0471630594. Translated under the title *Metod vozmushchenii*, Moscow, Mir Publ., 1976, 455 p.
4. Dyke M. van *Perturbation methods in fluid mechanics*. New York, London, Academic Press, 1964, 229 p. ISBN: 0127130500. Translated under the title *Metody vozmushchenii v mekhanike zhidkosti*, Moscow, Mir Publ., 1967, 310 p.
5. Il'in A.M. *Matching of asymptotic expansions of solutions of boundary value problems*. Providence, American Mathematical Society, 1992, 281 p. ISBN: 978-0-8218-4561-5. Original Russian text published in “Soglasovanie asimptoticheskikh razlozhenii reshenii kraevykh zadach”, Moscow, Nauka Publ., 1989, 336 p.
6. Lelikova E.F. On the asymptotics of a solution of a second order elliptic equation with small parameter at a higher derivative. *Proc. Steklov Inst. Math.* 2003. Suppl. 1, pp. S129–S143.
7. Lelikova E.F. The asymptotics of the solution of an equation with a small parameter in a domain with angular points. *Sbornik: Math.*, 2010, vol. 201, no. 10, pp. 1495–1510. doi: 10.1070/SM2010v201n10ABEH004119.
8. Lelikova E.F. On the asymptotics of a solution of a second order elliptic equation with a small parameter multiplying one of the highest order derivatives. *Trans. Moscow Math. Soc.*, 2010, vol. 71, pp. 141–174. ISSN: 1547-738X.
9. Lelikova E.F. On the asymptotic behavior of a solution to an equation with a small parameter in a neighborhood of a boundary inflection point. *Dokl. Math.*, 2012, vol. 86, no. 3, pp. 756–759.

10. Kondrat'ev V.A. Boundary value problems for elliptic equations in domains with conical or angular points. *Tr. Mosk. Mat. Obs.*, vol. 16, 1967, pp. 209–292 (in Russian).
11. Lelikova E.F. On the asymptotics of a solution to an equation with a small parameter in a neighborhood of a point of inflexion. *Tr. Inst. Mat. Mekh. UrO RAN*, 2016, vol. 22, no. 1, pp. 197–211 (in Russian).

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