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AN ANALYTIC METHOD FOR THE EMBEDDING OF THE EUCLIDEAN AND PSEUDO-EUCLIDEAN GEOMETRIES

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It is known that an n-dimensional geometry of maximum mobility admits a group of motions of dimension $n(n+1)/2$. Many of these geometries are well-known, for example, the Euclidean and pseudo-Euclidean geometries. Such geometries are phenomenologically symmetric; i.e., their metric properties are equivalent to their group properties. In this paper we consider the examples of the two-dimensional Euclidean and pseudo-Euclidean geometries to develop an analytical method for their embedding. More exactly, we search for all possible functions of the form $f = f((x_i - x_j)^2 \pm (y_i - y_j)^2, z_i, z_j)$, where, for example, x_i, y_i, z_i are the coordinates of a point i. It turns out that there exist only the following embeddings: $f = (x_i - x_j)^2 \pm (y_i - y_j)^2 + (z_i - z_j)^2$ and $f = [(x_i - x_j)^2 \pm (y_i - y_j)^2] \exp(2z_i + 2z_j)$. Note that we obtain not only the well-known three-dimensional geometries (Euclidean and pseudo-Euclidean) but also less known geometries (three-dimensional special extensions of the two-dimensional Euclidean and pseudo-Euclidean geometries). It is found that all these geometries admit six-dimensional groups of motions. To solve the formulated problem, according to the condition of local invariance of the metric function, we write the functional equation

$$2[(x_i - x_j)(X_1(i) - X_1(j)) + \epsilon(y_i - y_j)(X_2(i) - X_2(j))]\frac{\partial f}{\partial \theta} + X_3(i)\frac{\partial f}{\partial z_i} + X_3(j)\frac{\partial f}{\partial z_j} = 0,$$

where all the components are analytic functions. This equation is expanded in a Taylor series and the coefficients of the expansion at identical products of powers of the variables are compared. This task is greatly simplified by using the Maple 15 computing environment. The obtained results are used to write differential equations, which are then integrated to find solutions to the embedding problem formulated earlier.

Keywords: Euclidean geometry, functional equation, differential equation, metric function.

REFERENCES

1. Mikhailichenko G.G. Group and phenomenological symmetries in geometry. *Sib. Math. J.*, 1984, vol. 25, pp. 764–774.
2. Mikhailichenko G.G. *Polimetricheskie geometrii* [Polymetric geometries]. Novosibirsk: Novosibirskii Gos. Universitet Publ., 2001, 144 p.
3. Mikhailichenko G.G. Two-dimensional geometries. *Soviet Mathematics. Doklady*, 1981, vol. 24, pp. 346–348.
4. Lev V.Kh. Three-dimensional geometries in the theory of physical structures. *Vychislitel'nye sistemy*, Novosibirsk: Institute of Mathematics Publ., 1988, iss. 125, pp. 90–103 (in Russian).
5. Kyrov V.A. Functional equations in pseudo-Euclidean geometry. *Sib. Zh. Ind. Mat*, 2010, vol. 13, no. 4, pp. 38–51 (in Russian).

6. Kyrov V.A. Functional equations in symplectic geometry. *Trudy Inst. Mat. Mekh. UrO RAN*, 2010, vol. 16, no. 2, pp. 149–153 (in Russian).
7. Kyrov V.A. Embedding of phenomenologically symmetric geometries of two sets of the rank $(N, 2)$ into phenomenologically symmetric geometries of two sets of the rank (N + 1, 2). *Vestnik Udmurtskogo Universiteta. Matematika. Mekhanika. Komp'yuternye Nauki*, 2016, vol. 26, no. 3, pp. 312–323 (in Russian). doi: 10.20537/vm160302.
8. Ovsyannikov L.V. *Gruppovoi analiz differentsial'nykh uravnenii* [Group analysis of differential equations]. Moscow, Nauka Publ., 1978, 344 p.
9. Fikhtengol'ts G.M. *Kurs differentsial'nogo i integral'nogo ischisleniya* [A course in differential and integral calculus]. Moscow, Fizmatgiz Publ., 1963, vol. 2, 656 p.
10. D'yakonov V. *Maple* 10/11/12/13/14 *v matematicheskikh raschetakh* [Maple 10/11/12/13/14 in mathematical calculations]. Moscow: DMK-Press, 2014, 800 c. ISBN: 978-5-94074-751-2 .
11. Elsgolts L.E. *Differentsial'nye uravneniya i variatsionnoe ischislenie* [Differential equations and the calculus of variations]. Moscow: Nauka, 1969, 424 p.
12. Kyrov V.A. *Shestimernye algebry Li grupp dvizhenij trehmernyh fenomenologicheski simmetrichnyh geometrij* [Six-dimensional Lie algebras of the groups of motions of three-dimensional phenomenologically symmetric geometries], Appendix to the book G. G. Mikhailichenko, Polymetric geometries, Novosibirsk: Novosibirskii Gos. Universitet Publ., 2001, 144 p.

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