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ON THE POLYHEDRAL METHOD OF SOLVING PROBLEMS OF CONTROL STRATEGY SYNTHESIS IN DISCRETE-TIME SYSTEMS WITH UNCERTAINTIES AND STATE CONSTRAINTS

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We consider control synthesis problems for linear and bilinear discrete-time systems under uncertainties and state constraints. Two types of problems are studied: when controls are additive and when they appear in the system's matrix. For both problems we consider cases without uncertainty and cases with uncertainty, including additive parallelotope-bounded uncertainties and interval uncertainties in the matrix of the system. We continue to develop the methods of "polyhedral" control synthesis with the use of polyhedral (parallelotope-valued) solvability tubes. Namely, the technique proposed by the author earlier for solving the first problem is extended to the case of matrix uncertainties. Further, for both problems, a uniform solution scheme is developed, which makes it possible to construct control strategies by explicit formulas; this scheme extends another technique proposed earlier to the case of systems with state constraints. We describe polyhedral solvability tubes in the form of systems of nonlinear recurrence relations as well as control strategies that can be constructed with the use of these tubes. For the first problem, these two techniques produce identical polyhedral solubility tubes, but the control strategies turn out to be different; the relation between the controls is specified. Computer simulation results are presented.

Keywords: control synthesis, uncertainties, state constraints, solvability tubes, parallelotopes.

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