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VARIATIONAL PROBLEMS WITH UNILATERAL POINTWISE FUNCTIONAL CONSTRAINTS IN VARIABLE DOMAINS

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We consider a sequence of convex integral functionals $F_s\colon W^{1,p}(\Omega_s) \to \mathbb{R}$ and a sequence of weakly lower semicontinuous and, in general, non-integral functionals $G_s\colon W^{1,p}(\Omega_s) \to \mathbb{R}$, where $\{\Omega_s\}$ is a sequence of domains of $\mathbb{R}^n$ contained in a bounded domain $\Omega \subset \mathbb{R}^n$ $(n \geqslant 2)$ and $p > 1$. Along with this, we consider a sequence of closed convex sets $V_s = \{v \in W^{1,p}(\Omega_s)\colon v \geqslant K_s(v)$ a.e. in $\Omega_s\}$, where K_s is a mapping of the space $W^{1,p}(\Omega_s)$ into the set of all functions defined on Ω_s. We establish conditions under which minimizers and minimum values of the functionals $F_s + G_s$ on the sets V_s converge, respectively, to a minimizer and the minimum value of a functional on the set $V = \{v \in W^{1,p}(\Omega)\colon v \geqslant K(v)$ a.e. in $\Omega\}$, where K is a mapping of the space $W^{1,p}(\Omega)$ into the set of all functions defined on Ω. These conditions include, in particular, the strong connectedness of the spaces $W^{1,p}(\Omega_s)$ with the space $W^{1,p}(\Omega)$, the exhaustion condition of the domain Ω by the domains Ω_s, the Γ-convergence of the sequence $\{F_s\}$ to a functional $F\colon W^{1,p}(\Omega) \to \mathbb{R}$, and a certain convergence of the sequence $\{G_s\}$ to a functional $G\colon W^{1,p}(\Omega) \to \mathbb{R}$. We also assume certain conditions that characterize both the internal properties of the mappings K_s and their relation to the mapping K. In particular, these conditions admit the study of variational problems with unilateral varying irregular obstacles and with varying constraints combining the pointwise dependence and the functional dependence of the integral form.

Keywords: variable domains, integral functional, unilateral pointwise functional constraints, minimizer, minimum value, Γ-convergence, strong connectedness.

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