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PAINLEVE II EQUATION AS A MODEL OF A RESONANT INTERACTION OF OSCILLATORS

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We consider a system of differential equations that describes the interaction of two weakly connected nonlinear oscillators. The initial data are such that, if the connection is absent, the first oscillator is far from equilibrium and the second oscillator is near equilibrium; the eigenfrequencies of the oscillators are close to each other. The capture into resonance is investigated, when the frequencies of the connected oscillators remain close and the amplitudes of their oscillations undergo significant time variations; in particular, the second oscillator moves far from the equilibrium. We find that the initial stage of the resonance capture is described by a solution of the second Painleve equation. The description is obtained under an asymptotic approximation with respect to a small parameter corresponding to the connection factor.

Keywords: nonlinear equation, small parameter, asymptotics, oscillation, resonance.

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