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HIGH ACCURACY ALGORITHMS FOR APPROXIMATION OF DISCONTINUITY LINES OF A NOISY FUNCTION

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We consider the problem of localizing (finding the position of) discontinuity lines of a noisy function of two variables. Such problems arise in image processing, because the boundaries of objects are often discontinuity lines. It is assumed that the function of two variables is smooth in a neighborhood of discontinuity lines and has discontinuity of the first kind at each point of these lines. Instead of the exact function, its approximation in the space L_2 and the measurement error level δ are known. In this case, the problem belongs to the class of nonlinear ill-posed problems, and regularization algorithms should be constructed for its solution. We construct and study regularizing discrete algorithms of averaging "with a turn". New rules are proposed for choosing regularization parameters and the methods of deriving localization error bounds are improved. Error bounds are found for the localization of singularities of order $O(\delta^{4/3})$ under stricter separability conditions: the separability threshold in the present paper has order $O(\delta^{2/3})$, whereas in the authors' previous papers devoted to this problem the bounds for the localization error and separability threshold have order $O(\delta)$. In addition, the discretization of the algorithms of averaging "with a turn" is investigated theoretically for the first time (conditions on the discretization step are specified).

Keywords: ill-posed problem, regularization algorithm, localization of singularities, discontinuity of the first kind, discontinuity line.

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