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ON THE NUMERICAL SOLUTION OF DIFFERENTIAL GAMES FOR NEUTRAL-TYPE LINEAR SYSTEMS

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The paper deals with a zero-sum differential game, in which the dynamic of a conflict-controlled system is described by linear functional differential equations of neutral type and the quality index is the sum of two terms: the first term estimates the history of motion of the system realized by the terminal time, and the second term is an integral–quadratic estimation of the corresponding realizations of the players' controls. To calculate the value and construct the optimal control laws in this differential game, we propose an approach based on solving a suitable auxiliary differential game, in which the motion of a conflict-controlled system is described by ordinary differential equations and the quality index contains an estimation of the motion at the terminal time only. To find the value and the saddle point in the auxiliary differential game, we apply the so-called upper convex hull method, which leads to an effective solution in the case under consideration due to the specific structure of the quality index and the geometric constraints on the control actions of the players. The efficiency of the approach is illustrated by an example, and the results of numerical simulations are presented. The constructed optimal control laws are compared with the optimal control procedures with finite-dimensional approximating guides, which were developed by the authors earlier.

Keywords: differential games, neutral-type systems, optimal control strategies, numerical methods.

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