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A TWO-STAGE METHOD OF CONSTRUCTION OF REGULARIZING ALGORITHMS FOR NONLINEAR ILL-POSED PROBLEM

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For an equation with a nonlinear differentiable operator acting in a Hilbert space, we study a two-stage method of construction of a regularizing algorithm. First, we use Lavrientiev's regularization scheme. Then, we apply to the regularized equation either Newton's method or nonlinear analogs of α-processes: the minimum error method, the minimum residual method, and the steepest descent method. For these processes we establish the linear convergence rate and the Fejér property of iterations. Two cases are considered: when the operator of the problem is monotone and when the operator is finite-dimensional and its derivative has nonnegative spectrum. For the two-stage method with a monotone operator, we give an error bound, which has optimal order on the class of sourcewise representable solutions. In the second case, the error of the method is estimated by means of the residual. The proposed methods and their modified analogs are implemented numerically for three-dimensional inverse problems of gravimetry and magnetometry. The results of the numerical experiment are discussed.

Keywords: Lavrentiev regularization scheme, Newton's method, nonlinear α-processes, two-stage algorithm, inverse gravimetry and magnetometry problems.

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