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# STABILITY PROPERTIES OF THE VALUE FUNCTION IN AN INFINITE HORIZON OPTIMAL CONTROL PROBLEM

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Properties of the value function are examined in an infinite horizon optimal control problem with an integrand index appearing in the quality functional with a discount factor. The properties are analyzed in the case when the payoff functional of the control system includes a quality index represented by an unbounded function. An upper estimate is given for the growth rate of the value function. Necessary and sufficient conditions are obtained to ensure that the value function satisfies the infinitesimal stability properties. The question of coincidence of the value function with the generalized minimax solution of the Hamilton–Jacobi–Bellman–Isaacs equation is discussed. The uniqueness of the corresponding minimax solution is shown. The growth asymptotic behavior of the value function is described for the logarithmic, power, and exponential quality functionals, which arise in economic and financial modeling. The obtained results can be used to construct grid approximation methods for the value function as the generalized minimax solution of the Hamilton–Jacobi–Bellman–Isaacs equation. These methods are effective tools in the modeling of economic growth processes.

Keywords: optimal control, Hamilton–Jacobi equation, minimax solution, infinite horizon, value function, stability properties.

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