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## UPPER AND LOWER RESOLVING FUNCTIONS IN DYNAMIC GAME PROBLEMS

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The paper deals with game problems on the approach of trajectories of a nonstationary quasilinear system to a variable cylindrical terminal set. The case is studied when Pontryagin's classical condition fails. The notions of upper and lower resolving functions are introduced in the form of selections of special set-valued mappings. These functions are used to derive sufficient solvability conditions, which differ from the known ones. The results are illustrated with a model example.

Keywords: conflict-controlled process, set-valued mapping, Pontryagin's condition, Aumann's integral, resolving function.

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