A DISCRETE–CONTINUOUS ROUTING PROBLEM
WITH PRECEDENCE CONDITIONS

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We consider the problem of visiting closed sets in a compact metric space complicated by constraints in the form of precedence conditions and a possible dependence of the cost function on a list of tasks. We study a variant of the approximate realization of the extremum by applying models that involve problems of sequential visits to megalopolises (nonempty finite sets). This variant is naturally embedded into a more general construction that implements sequential visits to nonempty closed sets (NCSs) from a finite system in a metrizable compactum. The space of NCSs is equipped with the Hausdorff metric, which is used to estimate (under the corresponding condition that the sections of the cost functions are continuous) the proximity of the extrema in the problem of sequential visits for any two systems of NCSs (it is assumed that the numbers or NCSs in the systems are the same). The constraints in the form of precedence conditions are preserved.

Keywords: route, path, precedence conditions.

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