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OPTIMIZATION OF DYNAMICS OF A CONTROL SYSTEM IN THE PRESENCE OF RISK FACTORS

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The paper is concerned with the problem of optimization of dynamics of a control system in the situation when there is a set M ("risk zone") in the state space $\mathbb{R}^n$ which is unfavorable due to reasons of safety or instability of the system. In the classical setting the presence of such unfavorable set M is modeled usually via introducing an additional state constraint in the problem that means the ban on the presence of the trajectories in the risk zone M. Necessary optimality conditions in the form of Clarke's Hamiltonian inclusion are developed for the corresponding optimal control problem in the case when the system's dynamics is described by an autonomous differential inclusion and the risk zone M is an open set. The main novelty of the result is that it is proved in the most important case when the risk zone M is an open set. There is a natural relation of the problem under consideration to the classical optimal control problem with state constraints in this case. The result obtained involves an additional nonstandard stationarity condition for the Hamiltonian.

Keywords: risk zone, state constraints, optimal control, differential inclusion, Hamiltonian inclusion, Pontryagin maximum principle.

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