EXTERNAL ESTIMATES FOR REACHABLE SETS OF A CONTROL SYSTEM WITH UNCERTAINTY AND COMBINED NONLINEARITY

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The problem of estimating the trajectory tubes of a nonlinear control dynamic system with uncertainty in the initial data is studied. It is assumed that the dynamic system has a special structure in which the nonlinear terms are defined by quadratic forms in the state coordinates and the values of uncertain initial states and admissible controls are subject to ellipsoidal constraints. The matrix of the linear terms in the velocities of the system is not known exactly; it belongs to a given compact set in the corresponding space. Thus, the dynamics of the system is complicated by the presence of bilinear components in the right-hand sides of the differential equations of the system. We consider a complex case and generalize the author's earlier results. More exactly, we assume the simultaneous presence in the dynamics of the system of bilinear functions and quadratic forms (without the assumption of their positive definiteness), and we also take into account the uncertainty in the initial data and the impact of the control actions, which may also be treated here as undefined additive disturbances. The presence of all these factors greatly complicates the study of the problem and requires an adequate analysis, which constitutes the main purpose of this study. The paper presents algorithms for estimating the reachable sets of a nonlinear control system of this type. The results are illustrated by examples.

Keywords: control system, reachable set, state estimation, uncertainty.

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