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A LINEAR CONTROL PROBLEM UNDER INTERFERENCE WITH A PAYOFF DEPENDING ON THE MODULUS OF A LINEAR FUNCTION

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We consider a linear control problem in $\mathbb{R}^m$ under the action of an uncontrolled interference. The control process occurs on a given time interval $[t_0, p]$. The possible values of the interference belong to a compact set. The control is sought as the product of a scalar function $\phi(t) \in [\delta, \alpha]$ and a vector function $\xi(t, x) \in M$, $x \in \mathbb{R}^m$. The interval $[\delta, \alpha]$ and the convex symmetric compact set M are given. This definition of the control arises in control problems for mechanical systems of variable composition. For example, the law of variation of a reaction mass is defined as a function of time t, and the control affects the direction of relative velocity in which the mass is separated. The terminal part of the payoff depends on the modulus of a linear function of the vector $x(p)$. The integral part of the payoff is the integral over the interval $[t_0, p]$ of a given function $g(t, \phi(t))$, where $g(t, \phi) \geq 0$ for $t \in [t_0, p]$ and $\phi \in [\delta, \alpha]$. The control problem is considered within the theory of guaranteed result optimization. An optimal control existence theorem is proved under rather wide constraints on the class of problems. Sufficient conditions are found under which an admissible control is optimal. An example that illustrates the sufficient conditions is considered.

Keywords: control, interference, payoff, differential game.

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