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WEAK INVARIANCE OF A CYLINDRICAL SET WITH SMOOTH BOUNDARY WITH RESPECT TO A CONTROL SYSTEM

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A. A. Uspenskii

We consider the problem of constructing resolving sets for a differential game or an optimal control problem based on information on the dynamics of the system, control resources, and boundary conditions. The construction of largest possible sets with such properties (the maximal stable bridge in a differential game or the controllability set in a control problem) is a nontrivial problem due to their complicated geometry; in particular, the boundaries may be nonconvex and nonsmooth. In practical engineering tasks, which permit some tolerance and deviations, it is often admissible to construct a resolving set that is not maximal. The constructed set may possess certain characteristics that would make the formation of control actions easier. For example, the set may have convex sections or a smooth boundary. In this context, we study the property of stability (weak invariance) for one class of sets in the space of positions of a differential game. Using the notion of stability defect of a set introduced by V.N. Ushakov, we derive a criterion of weak invariance with respect to a conflict-controlled dynamic system for cylindrical sets. In a particular case of a linear control system, we obtain easily verified sufficient conditions of weak invariance for cylindrical sets with ellipsoidal sections. The proof of the conditions is based on constructions and facts of subdifferential calculation. An illustrating example is given.

Keywords: stable set, weak invariance, differential game, Hamiltonian, stability defect, cylindrical set, ellipsoid, subdifferential.

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Aleksandr Aleksandrovich Uspenskii, Cand. Sci. (Phys.-Math.), Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620990 Russia, e-mail: uspen@imm.uran.ru

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