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BILINEAR OPTIMAL CONTROL PROBLEM OF A DISCRETE LOGGIN

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In the proposed mathematical model a forest manager at each specified moment of time makes decisions on harvesting the trees of a certain type (species) and age (age group) in order to maximize their profit. When planning logging, the manager focuses on price projections and takes into account economic costs. The Pontryagin maximum principle is applied for solving the discrete-time optimal control problem arising in the model. A solution is derived in a constructive manner without computational costs associated with the problem's high-dimensionality. Analytical results, explaining the optimal solution, are provided. For a typically defined problem the optimality condition is derived, which determines the bang-bang solution. The condition includes the discrete dynamics of the adjoint variable, interpreted as the wood shadow price. The rule that is obtained is treated as the dynamic rationale for logging a certain type and age of forest. Structural flexibility of the proposed mathematical model facilitates its application in forest management. In proving theoretical results in the paper, the authors propose a method that they have not come across in the literature.

Keywords: Pontryagin's maximum principle, discrete forest management model.

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