MORPHOLOGICAL PROJECTOR IN THE L_0 METRIC AND THE PROBLEM OF LOCALIZATION OF STRUCTURAL DIFFERENCES BETWEEN IMAGES

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We consider the problem of localization of structural differences between two images given by Borel functions on a bounded planar set. For the case of finite-valued images, we propose a new algorithm for the calculation of the difference domain based on the morphological projection in the L_0 metric. It is shown that the algorithm gives an exact solution for a wide class of structural differences. It turned out that the algorithm based on the morphological projection in L_2 does not give an exact solution in the class of bounded structural changes. For the case of discrete images, when one of them is perturbed by a discrete independent normal white noise, we construct an algorithm for the calculation of the difference domain and show that the symmetric measure of the difference between the algorithm's output and the true difference set vanishes in probability under the unbounded growth of the ratio of the minimum jump to the standard deviation of the noise. We obtain a new estimate for the location of global maximum points for a Gaussian mixture of a special form.

Keywords: morphological analysis of images, morphological projector, Gaussian mixture, metric L_0, structural changes.

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