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ON THE SOLUTION OF A SYSTEM OF HAMILTON–JACOBI EQUATIONS OF SPECIAL FORM

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The paper is concerned with the investigation of a system of first-order Hamilton–Jacobi equations. We consider a strongly coupled hierarchical system: the first equation is independent of the second, and the Hamiltonian of the second equation depends on the gradient of the solution of the first equation. The system can be solved sequentially. The solution of the first equation is understood in the sense of the theory of minimax (viscosity) solutions and can be obtained with the help of the Lax–Hopf formula. The substitution of the solution of the first equation in the second Hamilton–Jacobi equation results in a Hamilton–Jacobi equation with discontinuous Hamiltonian. This equation is solved with the use of the idea of M-solutions proposed by A.I. Subbotin, and the solution is chosen from the class of set-valued mappings. Thus, the solution of the original system of Hamilton–Jacobi equations is the direct product of a single-valued and set-valued mappings, which satisfy the first and the second equations in the minimax and M-solution sense, respectively. In the case when the solution of the first equation is nondifferentiable only along one Rankine–Hugoniot line, existence and uniqueness theorems are proved. A representative formula for the solution of the system is obtained in terms of Cauchy characteristics. The properties of the solution and their dependence on the parameters of the problem are investigated.

Keywords: system of Hamilton–Jacobi equations, minimax solution, M-solution, Cauchy method of characteristics.

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