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ASYMPTOTICS OF THE SOLUTION TO THE SINGULAR PROBLEM OF OPTIMAL DISTRIBUTED CONTROL IN A CONVEX DOMAIN

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We consider the problem of optimal distributed control in a planar convex domain with smooth boundary and a small parameter at the highest derivatives of an elliptic operator. A zero Dirichlet condition is given at the boundary of the domain, and the control enters the inhomogeneity additively. The set of admissible controls is the unit ball in the corresponding space of square integrable functions. The solutions of the obtained boundary value problems are considered in the generalized sense as elements of some Hilbert space. The optimality index is the sum of the squared norm of the deviation of the state from a given state and the squared norm of the control with some coefficient. This structure of the optimality index makes it possible to strengthen, if necessary, the role of either the first or the second term of the index. In the first case it is more important to attain the desired state, whereas in the second case it is more important to minimize the resource consumption. We present a detailed study of the asymptotics of the problem generated by the sum of the Laplace operator with a small coefficient and a first-order differential operator. A special feature of the problem is the presence of characteristics of the limiting operator that are tangent to the boundary of the domain. We obtain a complete asymptotic expansion of the solution in powers of the small parameter in the case where the optimal control is an interior point of the set of admissible controls.

Keywords: singular problems, optimal control, boundary value problems for systems of partial differential equations, asymptotic expansions.

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