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ON EXTREMAL PROPERTIES OF THE BOUNDARY POINTS OF REACHABLE SETS FOR CONTROL SYSTEMS WITH INTEGRAL CONSTRAINTS

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It is well known that any control that steers the trajectory of a control system to the boundary of the reachable set satisfies the Pontryagin maximum principle. This fact is valid for systems with pointwise constraints on the control. We consider a system with quadratic integral constraints on the control. The system is nonlinear in the state variables and linear in the control. It is shown that any admissible control that steers the system to the boundary of its reachable set is a local solution of some optimal control problem with integral quadratic functional if the corresponding linearized system is completely controllable. The proof of this fact is based on the Graves theorem on covering mappings. This implies the maximum principle for the controls that steer the trajectories to the boundary of the reachable set. We also discuss an algorithm for constructing the reachable set based on the maximum principle.

Keywords: control system, integral constraints, reachable set, maximum principle.

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