A CONDITION FOR A FINITE GROUP TO BE A SCHMIDT GROUP[1]

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Let G be a finite group G, and let π be a set of primes such that $2 \in \pi$. We prove that if all maximal subgroups of G are π-closed and G itself is not π-closed then G is a Schmidt group. The proof employs the author's earlier results on the properties of pairs (G, π) where G is a simple minimal non-π-closed group and π is arbitrary.

Keywords: finite group, Schmidt group, π-closed group, simple group, maximal subgroup.

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