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ONE-SIDED INTEGRAL APPROXIMATIONS OF THE GENERALIZED POISSON KERNEL BY TRIGONOMETRIC POLYNOMIALS[1]

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We consider the generalized Poisson kernel $\Pi_{q,\alpha} = \cos(\alpha\pi/2)P + \sin(\alpha\pi/2)Q$ with $q \in (-1, 1)$ and $\alpha \in \mathbb{R}$, which is a linear combination of the Poisson kernel $P(t) = 1/2 + \sum_{k=1}^{\infty} q^k \cos kt$ and the conjugate Poisson kernel $Q(t) = \sum_{k=1}^{\infty} q^k \sin kt$. The values of the best upper and lower integral approximations of the kernel $\Pi_{q,\alpha}$ by trigonometric polynomials of order not exceeding a given number are found. The corresponding polynomials of the best one-sided approximation are obtained.

Keywords: constrained approximation, trigonometric polynomials, generalized Poisson kernel.

REFERENCES

1. Babenko A.G., Naum T.Z. One-sided approximations in L of a linear combination of the Poisson kernel and its conjugate kernel by trigonometric polynomials. *Proc. Internat. Summer Math. Stechkin School-Conf. on Function Theory*, Dushanbe: Polygraphy Ltd "Ofset", 2016, pp. 44–49 (in Russian).
2. Baraboshkina N.A. L-approximation of a linear combination of the Poisson kernel and its conjugate kernel by trigonometric polynomials. *Proc. Steklov Instit. Math*, 2011, vol. 273, suppl. 1, pp. S59–S67.
3. Baraboshkina N.A. Approximation of harmonic functions by algebraic polynomials on a circle of radius smaller than one with constraints on the unit circle. *Trudy Inst. Mat. Mekh. UrO RAN*, vol. 19, no. 2, 2013, pp. 71–78 (in Russian).
4. Bernstein S.N. Collected Works (Russian): Vol. 1: The constructive theory of functions (1905–1930), transl.: Atomic Energy Commission, Springfield, Va, 1958.
5. Bushanskii A.V. On the best harmonic approximation in the mean of some functions. *Investigations in the Theory of Approximation of Functions and Their Applications*, Institute of Mathematics Ukrainian Academy of Sciences, Kiev, 1978, pp. 29–37 (in Russian).
6. Zygmund A. *Trigonometric serie*, 2nd ed., New York: Cambridge University Press, 1959, Vol. 1,2.
7. Doronin V.G., Ligun A.A. Exact values of best one-sided approximations of certain classes of periodic functions. *Soviet Mathematics (Izvestiya VUZ. Matematika)*, 1979, vol. 23, no. 8, pp. 20–25.
8. Korneichuk N.P., Ligun A.A., Doronin V.G. *Approksimaciya s ogranicheniyami* (Approximation with Constraints). Kiev: Naukova Dumka, 1976, 252 p. (in Russian).
9. Krein M.G. On theory of best approximation of periodic functions. *Dokl. Akad. Nauk SSSR*, 1938, vol. 18, no. 4–5, pp. 245–249 (in Russian).
10. Chebyshev P.L. Problems about smallest quantities connected with an approximate representation of a function. *Complete Collected Works*, in 5 vol., Moscow: Izd. AN SSSR, 1947, vol. 2, pp. 151–235 (in Russian).
11. Bojanic R., DeVore R. On polynomials of best one-sided approximation. *Enseign. Math.*, 1966, vol. 12, pp. 139–164.

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12. Sz. Nagy B. Über gewisse Extremalfragen bei transformierten trigonometrischen Entwicklungen. I. Periodischer Fall, *Ber. Verh. sächs. Akad.*, Leipzig, 1938, Bd. 90, S. 103–134.
13. Stepanets A.I. Methods of approximation theory. Leiden, Boston: VSP, 2005, 919 p.

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