THE SET OF TARGET VECTORS IN A PROBLEM OF SEMI-INFINITE LINEAR PROGRAMMING WITH A DUALITY GAP[1]

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We propose a geometric method for the analysis of duality relations in a pair of semi-infinite linear programming (SILP) problems. The method is based on the use of the conical hull of the coefficients in the constraint system. A relation between the presence of a duality gap and the nonclosedness of the boundary of the conical hull of points in a multidimensional space is established. The geometric approach is used to construct an opposite pair of dual problems and to explore the duality relation for this pair. We construct a nontrivial example of a SILP problem in which the duality gap occurs for noncollinear target vectors.

Keywords: semi-infinite linear programming, duality gap, geometric approach, convex nonclosed cone, set of target vectors.

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