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THE JACKSON–STECHKIN INEQUALITY WITH NONCLASSICAL MODULUS OF CONTINUITY[1]

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We obtain an estimate for the best mean-square approximation $E_{n-1}(f)$ of an arbitrary complex-valued 2π-periodic function $f \in L_2$ by the subspace $\Im_{2n-1}$ of trigonometric polynomials of degree at most $n-1$ in terms of the nonclassical modulus of continuity $\omega^*_{2m-1}(f,\delta)_2$ generated by a finite-difference operator of order $2m-1$ with alternating constant coefficients equal to 1 in absolute value. The following relation is proved for any natural $n \geq 1$ and $m \geq 2$:

$$\sup_{\substack{f\in L_2\\ f\neq \mathrm{const}}} \frac{E_{n-1}(f)}{\Big(\frac{n}{2}\int_0^{\pi/n}\Big\{\omega^*_{2m-1}(f,t)\Big\}^2 \sin ntdt\Big)^{1/2}} = \frac{1}{\sqrt{2}}\Big(m-\sum_{l=1}^{m-1}\frac{l}{4(m-l)^2-1}\Big)^{-1/2}.$$

Keywords: best approximation, nonclassical modulus of continuity, Jackson–Stechkin inequality, convex function.

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