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MSC: 49K15, 91B62, 49J52

ON THE HAMILTONIAN IN INFINITE HORIZON CONTROL PROBLEMS¹

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We study necessary optimality conditions for infinite horizon control problems with performance functional containing a discounting factor of not necessarily exponential form. A uniformly overtaking criterion is considered as an optimality criterion. The behavior of the pair “adjoint variable–Hamiltonian” at infinity in a neighborhood of an optimal trajectory is described in terms of limiting gradients of the payoff function. This guarantees the existence of the limiting solution of the Pontryagin maximum principle corresponding to the optimal process. We discuss the assumptions that provide the necessity of both a condition of the Michel type for the maximized Hamiltonian and a formula of the Cauchy type for the adjoint variable proposed in papers by Kryazhimskii and Aseev. In particular, this extends the maximum principle to a complete system of relations. The case of a discounting factor of the form $(1 + t)^{-s}$ is considered separately.

Keywords: infinite horizon control problem, necessary conditions, transversality condition at infinity, Pontryagin maximum principle, Michel condition, uniformly overtaking optimal control.

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