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## OPTIMAL RECOVERY OF A FUNCTION ANALYTIC IN A DISK FROM APPROXIMATELY GIVEN VALUES ON A PART OF THE BOUNDARY<sup>1</sup>

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We study three related extremal problems in the space  $\mathcal{H}$  of functions analytic in the unit disk such that their boundary values on a part  $\gamma_1$  of the unit circle  $\Gamma$  belong to the space  $L_{\psi_1}^\infty(\gamma_1)$  of functions essentially bounded on  $\gamma_1$  with weight  $\psi_1$  and their boundary values on the set  $\gamma_0 = \Gamma \setminus \gamma_1$  belong to the space  $L_{\psi_0}^\infty(\gamma_0)$  with weight  $\psi_0$ . More exactly, on the class  $Q$  of functions from  $\mathcal{H}$  such that the norm  $L_{\psi_0}^\infty(\gamma_0)$  of their boundary values on  $\gamma_0$  does not exceed one, we solve the problem of optimal recovery of an analytic function on a subset of the unit disk from its boundary values on  $\gamma_1$  specified approximately with respect to the norm  $L_{\psi_1}^\infty(\gamma_1)$ . We also study the problem of the optimal choice of the set  $\gamma_1$  under a given fixed value of its measure. The problem of the best approximation of the operator of analytic continuation from a part of the boundary by linear bounded operators is investigated.

Keywords: optimal recovery of analytic functions, best approximation of unbounded operators, Szegő function.

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