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INTERPOLATION WAVELETS IN BOUNDARY VALUE PROBLEMS[1]

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We propose and validate a simple numerical method that finds an approximate solution with any given accuracy to the Dirichlet boundary value problem in a disk for a homogeneous equation with the Laplace operator. There are many known numerical methods that solve this problem, starting with the approximate calculation of the Poisson integral, which gives an exact representation of the solution inside the disk in terms of the boundary values of the required functions. We employ the idea of approximating a given 2π-periodic boundary function by trigonometric polynomials, since it is easy to extend them to harmonic polynomials inside the disk so that the deviation from the required harmonic function does not exceed the error of approximation of the boundary function. The approximating trigonometric polynomials are constructed by means of an interpolation projection to subspaces of a multiresolution analysis (approximation) with basis 2π-periodic scaling functions (more exactly, their binary rational compressions and shifts). Such functions were constructed by the authors earlier on the basis of Meyer-type wavelets; they are either orthogonal and at the same time interpolational on uniform grids of the corresponding scale or only interpolational. The bounds for the rate of approximation of the solution to the boundary value problem are based on the property of Meyer wavelets to preserve trigonometric polynomials of certain (large) orders; this property was used for other purposes in the first two papers from the list of references. Since a numerical bound of the approximation error is very important for the practical application of the method, a considerable portion of the paper is devoted to this issue, more exactly, to the explicit calculation of the constants in the order bounds of the error known earlier.

Keywords: wavelets, interpolation wavelets, harmonic function, Dirichlet problem.

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