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ON HANKEL OPERATORS ASSOCIATED WITH LINEARLY ORDERED ABELIAN GROUPS[1]

A. R. Mirotin, E. Yu. Kuz'menkova

We consider two variants of generalizations of Hankel operators to the case of linearly ordered abelian groups. Criteria for the boundedness and compactness of these operators are given, in particular, in terms of functions of bounded mean oscillation. It is proved that the generalized Hankel operators are non-Fredholm. Some applications to the theory of Toeplitz operators on groups are given.

Keywords: Hankel operator, integral Hankel operator, Fredholm operator, compact operator, bounded mean oscillation, linearly ordered abelian group, compact abelian group, Toeplitz operator.

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A. R. Mirotin, Dr. Phys.-Math. Sci, Prof., Francisk Skorina Gomel State University, Gomel, 246028 Belarus, e-mail: amirotin@yandex.ru .

E. Yu. Kuz'menkova, doctoral student, Francisk Skorina Gomel State University, Gomel, 246028 Belarus, e-mail: katuha66@tut.by .

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