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ON SOME FUNCTIONS OF A LINEAR CLOSED OPERATOR¹

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A densely defined linear injective operator whose regular set contains the negative real semiaxis is given in a complex Banach space. A power asymptotic bound is known for the norm of the resolvent of this operator at zero and at infinity on the same semiaxis. We study some classes of functions of this operator constructed (in view of the Cauchy integral formula) on the basis of the corresponding scalar analytic functions that have power asymptotic bounds for their moduli at zero and at infinity. A number of properties of the operator functions are established, in particular, the multiplicative property and the invertibility. It is proved that a linear combination of integer powers of an arbitrary linear injective operator with nonempty resolvent set (under natural constraints on the coefficients) is a closed operator and that functions of an operator A constructed for a linear combination of scalar power functions with integer exponents coincide with the corresponding linear combination of powers of this operator.

Keywords: linear closed operator, functions of an operator, multiplicative property, invertibility.

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