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THE INVERSE THEOREM IN VARIOUS METRICS OF APPROXIMATION THEORY FOR PERIODIC FUNCTIONS WITH MONOTONE FOURIER COEFFICIENTS [1]

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We prove the exactness with respect to order of an upper bound for the kth-order modulus of smoothness in $L_q(\mathbb{T})$ in terms of the elements of a sequence of best approximations in $L_p(\mathbb{T})$ on the class of all functions with monotonically decreasing Fourier coefficients, where $1 < p < q < \infty$ and $k \in \mathbb{N}$.

Keywords: modulus of smoothness, best approximation, inverse theorem in various metrics, trigonometric Fourier series with monotone coefficients, order-sharp inequality on a class.

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