

DOI: 10.21538/0134-4889-2016-22-4-136-152

MSC: 34B24, 41A44, 41A55, 41A63

**APPROXIMATION IN L_2 BY PARTIAL INTEGRALS
OF THE MULTIDIMENSIONAL FOURIER TRANSFORM
IN THE EIGENFUNCTIONS OF THE STURM–LIOUVILLE OPERATOR¹**

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For approximations in the space $L^2(\mathbb{R}_+^d)$ by partial integrals of the multidimensional Fourier transform in the eigenfunctions of the Sturm–Liouville operator, we prove the Jackson inequality with exact constant and optimal argument in the modulus of continuity. The multidimensional weight that defines the Sturm–Liouville operator is the product of one-dimensional weights. The one-dimensional weights can be, in particular, power and hyperbolic weights with various parameters. The optimality of the argument in the modulus of continuity is established by means of the multidimensional Gauss quadrature formula over zeros of an eigenfunction of the Sturm–Liouville operator. The obtained results are complete; they generalize a number of known results.

Keywords: Sturm–Liouville operator, L^2 -space, Fourier transform, Jackson inequality, Gauss quadrature formula.

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¹Received July 30, 2016

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Cite this article as:

D. V. Gorbachev, V. I. Ivanov, R. A. Veprintsev, Approximation in L_2 by partial integrals of the multidimensional Fourier transform in the eigenfunctions of the Sturm–Liouville operator, *Trudy Inst. Mat. Mekh. UrO RAN*, 2016, vol. 22, no. 4, pp. 136–152 .