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On APPROXIMATION ORDERS OF FUNCTIONS OF SEVERAL VARIABLES IN THE LORENTZ SPACE¹

G. Akishev

We consider the anisotropic Lorentz space of periodic functions. Sufficient conditions are proved for a function to belong to the anisotropic Lorentz space. Estimates for the order of approximation by trigonometric polynomials of the Nikol'skii–Besov class in the anisotropic Lorentz space are established.

Keywords: Lorentz space, Nikol'skii–Besov class, best approximation.

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G. Akishev, Dr. Phys.-Math. Sci., Prof., RSE Academician E.A. Buketov Karaganda State University, the Republic of Kazakhstan, 100028; Institute of Mathematics and Computer Science, Ural Federal University, Yekaterinburg, 620002 Russia, e-mail: akishev_g@mail.ru .

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