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### ON DEZA GRAPHS WITH DISCONNECTED SECOND NEIGHBORHOOD OF A VERTEX

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A graph  $\Gamma$  is called a Deza graph if it is regular and the number of common neighbors of two distinct vertices is one of two values. A Deza graph  $\Gamma$  is called a strictly Deza graph if it has diameter 2 and is not strongly regular. In 1992, Gardiner, Godsil, Hensel, and Royle proved that a strongly regular graph that contains a vertex with disconnected second neighborhood is a complete multipartite graph with parts of the same size and this size is greater than 2. In this paper we study strictly Deza graphs with disconnected second neighborhoods of vertices. In Section 2, we prove that, if each vertex of a strictly Deza graph has disconnected second neighborhood, then the graph is either edge-regular or coedge-regular. In Sections 3 and 4, we consider strictly Deza graphs that contain at least one vertex with disconnected second neighborhood. In Section 3, we show that, if such a graph is edge-regular, then it is an  $s$ -coclique extension of a strongly regular graph with parameters  $(n, k, \lambda, \mu)$ , where  $s$  is integer,  $s \geq 2$ , and  $\lambda = \mu$ . In Section 4, we show that, if such a graph is coedge-regular, then it is a 2-clique extension of a complete multipartite graph with parts of the same size greater than or equal to 3.

Keywords: Deza graph, strictly Deza graph, disconnected second neighborhood, edge-regular graph, coedge-regular graph.

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