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APPROXIMATION SCHEMES FOR THE GENERALIZED TSP

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The generalized traveling salesman problem (GTSP) is defined by a weighted graph $G = (V, E, w)$ and a partition of its vertex set into k disjoint clusters $V = V_1 \cup \ldots \cup V_k$. It is required to find a minimum-weight cycle that contains exactly one vertex of each cluster. We consider a geometric setting of the problem (we call it EGTSP-k-GC), in which the vertices of the graph are points in a plane, the weight function corresponds to the euclidian distances between the points, and the partition into clusters is specified implicitly by means of a regular integer grid with step 1. In this setting, a cluster is a subset of vertices lying in the same cell of the grid; the arising ambiguity is resolved arbitrarily. Even in this special setting, the GTSP remains intractable, generalizing in a natural way the classical planar Euclidean TSP. Recently, a $(1.5+8\sqrt{2}+\varepsilon)$-approximation algorithm with complexity depending polynomially both on the number of vertices n and on the number of clusters k has been constructed for this problem. We propose three approximation algorithms for the same problem. For any fixed k, all the schemes are PTAS and the complexity of the first two is linear in the number of nodes. Furthermore, the complexity of the first two schemes remains polynomial for $k = O(\log n)$, whereas the third scheme is polynomial for $k = n - O(\log n)$.

Keywords: generalized traveling salesman problem, NP-hard problem, polynomial-time approximation scheme.

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