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**A CRITERION FOR THE FAILURE OF LOCAL BALANCE
OF SOME SIMPLE GROUPS OF LIE TYPE**

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A finite simple nonabelian group K is called locally balanced (locally 1-balanced) with respect to a prime p if $O_{p'}(C_G(x)) = 1$ for any element x of order p from $G \simeq \text{Aut}(K)$. Finite simple nonabelian groups that are not locally balanced were described in the famous Theorem 7.7.1 from *The Classification of the Finite Simple Groups* by Gorenstein, Lyons, and Solomon. However, there is a gap in statement (d) of that theorem, which is also present in the proof. In this connection, we prove the following theorem.

Theorem. *Suppose that G is a finite almost simple group, $K = \text{Soc}(G)$ is a group of Lie type over a field of characteristic r , and x is an element of a prime order $p \neq r$ from G that induces on K a non-inner-diagonal automorphism. Then the following conditions are equivalent:*

- (1) $O_{p'}(C_G(x)) \neq 1$;
- (2) x induces a field automorphism on K and $(|C_K(x)|, p) = 1$.

The theorem gives a criterion for the local 1-imbalance of groups of Lie type from statement (d) of the mentioned Theorem 7.7.1 with a non-inner-diagonal automorphism. The criterion can be used to construct a countable series of counterexamples to this statement for any simple nonabelian group of Lie type.

Keywords: finite group, simple group, group of Lie type, balanced group.

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