

**A PRONORMALITY CRITERION FOR SUPPLEMENTS TO ABELIAN
NORMAL SUBGROUPS**

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A subgroup H of a group G is called pronormal if, for any element $g \in G$, the subgroups H and H^g are conjugate in the subgroup $\langle H, H^g \rangle$. We prove that, if a group G has a normal abelian subgroup V and a subgroup H such that $G = HV$, then H is pronormal in G if and only if $U = N_V(H)[H, U]$ for any H -invariant subgroup U of the group V . Using this fact, we prove that the simple symplectic group $\mathrm{PSp}_{6n}(q)$ with $q \equiv \pm 3 \pmod{8}$ contains a nonpronormal subgroup of odd index. Hence, we disprove the conjecture on the pronormality of subgroups of odd indices in finite simple groups, which was formulated in 2012 by E.P. Vdovin and D.O. Revin and verified by the authors in 2015 for many families of simple finite groups.

Keywords: pronormal subgroup, complement of a subgroup, supplement of a subgroup, finite simple group, subgroup of odd index.

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