

**ESTIMATES FOR MEAN-SQUARE NORMS OF FUNCTIONS WITH  
LACUNARY FOURIER SERIES****A. G. Babenko, V. A. Yudin**

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We consider the properties of functions  $f$  from the space  $L^2(\mathbb{T})$  on the period  $\mathbb{T} = [-\pi, \pi)$  with lacunary Fourier series such that the size of each lack is not less than a given positive integer  $q - 1$ . We find two-sided estimates of the  $L^2$  norms of such functions on  $\mathbb{T}$  in terms of similar norms (more exactly, seminorms) on intervals  $I$  of length  $|I| = 2h < 2\pi$ . The estimates are obtained in terms of best one-sided integral approximations of the characteristic function of the interval  $(-h, h)$  by trigonometric polynomials of order at most  $q - 1$ . The issue considered in this paper appeared first in N. Wiener's studies (1934). Important results in this area were obtained by A.E. Ingham (1936) and by A. Selberg in the 1970s.

Keywords: lacunary trigonometric series, mean-square norms, one-sided approximation of functions by trigonometric polynomials.

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