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DERIVATIVES BY VIRTUE OF DIFFEOMORPHISMS AND THEIR APPLICATIONS IN CONTROL THEORY AND GEOMETRICAL OPTICS

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The paper deals with nonsmooth problems of optimal control theory and geometrical optics that can be formalized as Dirichlet boundary value problems for first-order partial differential equations (including equations of Hamiltonian type). A methodology is elaborated for the identification and construction of singular sets with the use of multipoint derivatives. Four types of derivatives by virtue of diffeomorphisms are introduced; they generalize the notions of classical derivative and one-sided derivative. Formulas are given for the calculation of derivatives by virtue of diffeomorphisms for some classes of functions. The efficiency of the developed method of analysis is illustrated by the example of solving an optimal time problem in the case of a circular velocity vectogram and nonconvex target with nonsmooth boundary.

Keywords: first-order PDE, minimax solution, wavefront, diffeomorphism, eikonal, optimal result function, singular set, symmetry.

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