TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 21 No. 1 **2015**

DIFFERENCE SCHEME OF HIGHEST ACCURACY ORDER FOR A SINGULARLY PERTURBED REACTION–DIFFUSION EQUATION BASED ON THE SOLUTION DECOMPOSITION METHOD

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Received December 15, 2014

A Dirichlet problem is considered for a singularly perturbed ordinary differential reaction–diffusion equation. For this problem, a new approach is developed in order to construct difference schemes whose solutions converge in the maximum norm uniformly with respect to the perturbation parameter ε, $\varepsilon \in (0,1]$ (i.e., ε-uniformly) with order of accuracy significantly greater than the achievable accuracy order for the Richardson method on piecewise-uniform grids. Important in this approach is the use of uniform grids for solving grid subproblems for regular and singular components of the grid solution. Using the asymptotic construction technique, a basic difference scheme of the solution decomposition method is constructed that converges ε-uniformly in the maximum norm at the rate $\mathcal{O}\left(N^{-2}\ln^2 N\right)$, where $N+1$ is the number of nodes in the uniform grids used. The Richardson extrapolation technique on three embedded grids is applied to the basic scheme of the solution decomposition method. As a result, we have constructed the Richardson scheme of the solution decomposition method with highest accuracy order. The solution of this scheme converges ε-uniformly in the maximum norm at the rate $\mathcal{O}\left(N^{-6}\ln^6 N\right)$.

Keywords: singularly perturbed boundary value problem, ordinary differential reaction–diffusion equation, decomposition of a discrete solution, asymptotic construction technique, difference scheme of the solution decomposition method, uniform grids, ε-uniform convergence, maximum norm, Richardson extrapolation technique, difference scheme of highest accuracy order.

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Cite this article as:
G. I. Shishkin, L. P. Shishkina, Difference scheme of highest accuracy order for a singularly perturbed reaction–diffusion equation based on the solution decomposition method, *Tr. Inst. Mat. Mekh. UrO RAN*, 2015, vol. 21, no. 1, pp. 280–293 .