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ON PIECEWISE CONSTANT APPROXIMATION IN DISTRIBUTED OPTIMIZATION PROBLEMS

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The paper is devoted to optimal control problems for distributed parameter systems representable by functional operator equations of Hammerstein type in a Banach space compactly embedded in a Lebesgue space. The problem of minimizing an integral functional on a set of "state–control" pairs satisfying a control equation of the mentioned type is considered. We prove that this problem is equivalent to an optimization problem obtained from the original one by passing to a description of the control system in terms of V.I. Sumin's functional operator equation in a Lebesgue space. The equivalent optimization problem is called S-dual. For an S-dual optimization problem, we investigate a piecewise constant approximation for the "state–control" pair. For this approximation method, we state the following results: (1) convergence of piecewise constant approximations with respect to the functional and the equation for the S-dual optimization problem; (2) existence of a global solution of an approximating finite-dimensional mathematical programming problem; (3) convergence with respect to the functional of solutions of an approximating optimization problem to a solution of the original problem. As an auxiliary result of independent interest, we prove a theorem on the total (over the whole set of admissible controls) preservation of solvability for a control equation of Hammerstein type. The Dirichlet problem for a semilinear elliptic equation of diffusion–reaction type is considered as an example of reducing a distributed parameter control system to such an equation.

Keywords: piecewise constant approximation, optimal control, equation of Hammerstein type, convergence by functional, total preservation of solvability, semilinear stationary diffusion–reaction equation.

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